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Coherence dynamics in microcavities and polariton lasers

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Abstract

We study theoretically the second-order coherence $g^{(2)}(0)$ of light emitted by polariton lasers, i.e., devices based on stimulated relaxation and condensation of exciton–polaritons in microcavities. We solve kinetic equations for the polaritons in different approximations and show that (i) the coherence introduced into the polariton condensate by an external source can be conserved by the system over a macroscopically long time, and (ii) if the total number of polaritons is fixed by the excitation conditions, the correlations between the populations of the ground and excited polariton states can also result in the spontaneous buildup of second-order coherence in the polariton condensate. Both results are obtained neglecting polariton–polariton interactions in the condensate.

1. Introduction

In this paper we discuss the so-called *polariton laser*, a promising device to exhibit various coherent phenomena at room temperature, namely, lasing, Bose–Einstein condensation (BEC) and superfluidity. A polariton laser is effectively represented by a microcavity in which the confined photon mode is put in resonance with an excitonic transition. If the coupling strength between excitons and photons is large enough, new quasiparticles appear, which are called exciton–polaritons (or polaritons for short), displaying boson properties while having a very light effective mass. Being Bose condensed, the exciton–polaritons are expected to spontaneously emit coherent and monochromatic light which represents the polariton laser effect. We address theoretically the coherence of light emitted by such a structure and investigate, in particular, the possibility of its buildup in the polariton field. This work has been motivated by numerous experimental results published in recent years [1–11]. To

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summarize them, polariton lasing has not been experimentally evidenced yet, while many bosonic effects like the stimulated scattering of polaritons and the resulting amplification of radiation of microcavities have been demonstrated. As revealed in many works, one specificity of polariton physics in its relation to coherence is linked to the ultrafast relaxation dynamics of exciton–polaritons in microcavities. It has both advantages, for example in regard to its technological applications, and disadvantages, because it makes it harder to achieve coherence experimentally and more complicated to describe it theoretically.

This paper is organized as follows. The polariton laser and its unique predispositions to house coherent phenomena are presented in section 2.1 along with the difficulties posed by the dynamics. As was demonstrated experimentally, the device can be brought to operate in some regimes where coherent effects and bosonic statistics are ruling the carriers kinetics. First dynamical descriptions put forward the semi-classical Boltzmann equation, i.e., an equation of motion for the distribution functions $n_{\mathbf{k}}$ (the average number of polaritons with the wavevector $\mathbf{k}$), where transition rates are enhanced by a factor $(1 + n_{\mathbf{k}})$ to take into account bosonic stimulation. This picture, reviewed in section 2.2, captures the essential features of the problem. In section 2.3 we present the extension of the Boltzmann formalism which enables the description of the quantum characteristics of the polariton condensate, as for instance its coherence degree. While the Boltzmann approach cannot describe the spontaneous coherence formation, we show that if a small coherent seed of polaritons is introduced in the ground state, this coherence can survive and is accompanied by the ground-state population buildup. The coherence can also be essentially introduced in the polariton system by the experimental conditions, in particular, when the number of polaritons is fixed by a pulsed excitation, in which case it requires no seed. In section 3 we study how this coherence can be transferred into the polariton condensate in the case of fast relaxation. Throughout the paper we assume nonresonant optical pumping which creates electron–hole pairs in the quantum well embedded in a microcavity. We assume that these electrons and holes quickly relax in energy, forming excitons that immediately couple to the photonic mode of the cavity. We work exclusively in the strong-coupling regime where the two exciton–polariton branches are split by the so-called Rabi-splitting of about 10 meV, and we neglect any dependence of the dispersion curves of exciton–polaritons on their concentration. We properly account for the energy and wavevector relaxation along the lower polariton branch, assisted by acoustic phonons or caused by polariton–polariton scattering. Also we account for the finite lifetime of exciton–polaritons which represents their fundamental difference from other kinds of interacting bosons (atoms, Cooper pairs, etc). We do not presently discuss any effects linked to the spin or dipole moment dynamics of exciton polaritons. Our aim here is to study under the above assumptions the dynamics of quantum coherence of the lowest energy polariton state in order to see if it can be conserved on a longer timescale than the polariton lifetime and if it can build up spontaneously (in the absence of any seed). These two questions to which we find positive answers are of fundamental importance for the application of polariton lasers as optical memory elements and for evidencing Bose condensation of exciton polaritons.

2. Microcavity polaritons and their kinetics

2.1. Microcavity polaritons: strong coupling of excitons and photons

The coupling of excitons and photons in conventional quantum wells is weak—in the sense that it cannot be retained for long times—because coupling (of the dipole–field type) occurs for equal wavevectors, and for a single 2D exciton there is a continuum of 3D photons with the wavevector projection in the plane matching that of the exciton. Coupling of a single state to

a continuum leads to irreversible dissipation of the single state energy into the continuum. A one-to-one coupling can be restored by confining the photons, which can be done by placing the quantum well (QW) inside a Fabry–Perot-like microcavity, effectively made up of many alternating semiconductors of varying refractive indices.

A quantum theory of such a structure should start with the microscopic Hamiltonian expressed in terms of $e_{\mathbf{k}}$, $h_{\mathbf{k}}$ and $c_{\mathbf{k}}$, the second-quantized annihilation operators of electrons, holes and photons, respectively, in the basis of 2D-plane waves for which the in-plane wavevector $\mathbf{k}$ is the good quantum number. At this point, we do not need the exact form of this Hamiltonian, which provides the foundations for semiconductor Bloch equations, of general validity (see for instance [12, 13]). It suffices to recall that in the low density limit in which we shall place ourselves for the remainder of this text, this Hamiltonian can be diagonalized by introducing the exciton operator $b_{\mathbf{k}}$ defined by

$$b_{\mathbf{k}} \equiv \sum_{\mathbf{q}} \psi(\mathbf{q}) e_{\mathbf{q}+\frac{1}{2}\mathbf{k}} h_{-\mathbf{q}+\frac{1}{2}\mathbf{k}}, \tag{1}$$

where ψ is the Fourier transform of the bound state wavefunction (we neglect excited states in the confined direction). In terms of this operator we obtain in some approximations [13] the exciton–photon Hamiltonian (neglecting spin degree of freedom):

$$H = \sum_{\mathbf{k}} E_{\mathrm{ex}}(\mathbf{k}) b_{\mathbf{k}}^{\dagger} b_{\mathbf{k}} + \sum_{\mathbf{k}} E_{\gamma}(\mathbf{k}) c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \sum_{\mathbf{k}} \frac{\hbar\Omega_{\mathrm{R}}}{2} (b_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + b_{\mathbf{k}} c_{\mathbf{k}}^{\dagger}), \tag{2}$$

with the sum being taken over the denumerable set of plane wave solutions in a 2D-box. Here $E_{\mathrm{ex}}(\mathbf{k})$ is the dispersion relation of the exciton:

$$E_{\mathrm{ex}}(\mathbf{k}) = E_{\mathrm{ex}}(0) + \frac{\hbar^2 k^2}{2m_{\mathrm{ex}}}, \tag{3}$$

where $E_{\mathrm{ex}}(0)$ is the rest energy of the bound state hole–electron, and m_{ex} is the effective mass, so big that the dispersion is almost flat over the range of wavevectors of interest. E_{γ} is the dispersion of the confined photon,

$$E_{\gamma}(\mathbf{k}) = \sqrt{E_{\gamma}(0) + \frac{\hbar^2 c^2 k^2}{n^2}}, \tag{4}$$

where c is the speed of light and n is the refractive index of the cavity. Because of $E_{\gamma}(0) \equiv \hbar^2 c^2 k_{\perp}^2 / n^2$, with $k_{\perp}$ being the part of the wavevector quantized by the confinement, the dispersion relation of the 2D-photon is not the usual linear one but is close to a parabola, providing it with a small effective mass $E_{\gamma}(0) n^2 / c^2$. As for the exciton–photon coupling (the last term in equation (2)), we simply write its strength as $\hbar\Omega_{\mathrm{R}}/2$ with Ω_{R} being dependent on the exciton oscillator strength and the quality factor of the cavity [13, 14]. This properly describes microcavities in the linear regime, and in particular it gives an equivalent description of the semiclassical theory written for the Maxwell fields with a Lorentzian dielectric function to describe the exciton. Equation (2), however, neglects such effects as the exciton coupling to the continuum of unconfined photon states outside the cavity [14]. It can nevertheless be shown to be exact close to resonance [13].

The photon is a boson and thus $c_{\mathbf{k}}$ obeys bosonic algebra, $[c_{\mathbf{k}}, c_{\mathbf{q}}^{\dagger}] = \delta_{\mathbf{k},\mathbf{q}}$ with all other commutators zero. However, with the fermion (anticommutating) algebra for e and h operators, the commutator for $b_{\mathbf{k}}$ as defined by (1) reads

$$\begin{aligned}[b_{\mathbf{k}}, b_{\mathbf{k}}^{\dagger}] &= \sum_{\mathbf{q},\mathbf{q}'} \psi(\mathbf{q}) \psi(\mathbf{q}')^{*} \left[e_{\mathbf{q}+\frac{1}{2}\mathbf{k}} h_{-\mathbf{q}+\frac{1}{2}\mathbf{k}}, e_{\mathbf{q}'+\frac{1}{2}\mathbf{k}}^{\dagger} h_{-\mathbf{q}'+\frac{1}{2}\mathbf{k}}^{\dagger} \right] \\ &= 1 - \sum_{\mathbf{q}} |\psi(\mathbf{q})|^2 \left(e_{\mathbf{q}+\frac{1}{2}\mathbf{k}}^{\dagger} e_{\mathbf{q}+\frac{1}{2}\mathbf{k}} - h_{-\mathbf{q}+\frac{1}{2}\mathbf{k}}^{\dagger} h_{-\mathbf{q}+\frac{1}{2}\mathbf{k}} \right), \end{aligned} \tag{5}$$

and is therefore bosonic only in the limit of vanishing number occupancy, i.e., in the low density limit. Especially, $\langle [b_0, b_0^\dagger] \rangle = 1 - \mathrm{O}(na_0^2)$, where n is the density of excitons and a_0 is the Bohr radius associated to ψ. One can therefore treat excitons as bosons only in the limit $na_0^2 \ll 1$. In such an approximation, the Hamiltonian (2), bilinear in bosonic operators, can be diagonalized with the Hopfield transformation:

$$a_{\mathbf{k}}^{\mathrm{L}} \equiv X_{\mathbf{k}} b_{\mathbf{k}} - C_{\mathbf{k}} c_{\mathbf{k}}, \tag{6}$$

$$a_{\mathbf{k}}^{\mathrm{U}} \equiv C_{\mathbf{k}} b_{\mathbf{k}} + X_{\mathbf{k}} c_{\mathbf{k}}, \tag{7}$$

where the coefficients $C_{\mathbf{k}}$ and $X_{\mathbf{k}}$ satisfy $C_{\mathbf{k}}^2 + X_{\mathbf{k}}^2 = 1$, so that the transformation is canonical and a operators follow the bosonic algebra as well. The Hamiltonian (2) now reduces to free propagation terms only:

$$H = \sum_{\mathbf{k}} E_{\mathrm{U}}(\mathbf{k}) a_{\mathbf{k}}^{\mathrm{U}\dagger} a_{\mathbf{k}}^{\mathrm{U}} + \sum_{\mathbf{k}} E_{\mathrm{L}}(\mathbf{k}) a_{\mathbf{k}}^{\mathrm{L}\dagger} a_{\mathbf{k}}^{\mathrm{L}}, \tag{8}$$

for *upper* and *lower polariton branches*, with second quantized annihilation operators a^{U} and a^{L}, respectively. The dispersion relations for these branches are

$$E_{\substack{\mathrm{U} \\ \mathrm{L}}}(\mathbf{k}) = \tfrac{1}{2}(E_{\mathrm{ex}}(\mathbf{k}) + E_{\gamma}(\mathbf{k})) \pm \tfrac{1}{2}\sqrt{\delta_{\mathbf{k}}^2 + \hbar^2 \Omega_{\mathrm{R}}^2}, \tag{9}$$

(the U subscript is associated with the plus sign, L with minus) where $\delta_{\mathbf{k}}$ is the energy mismatch, or *detuning*, between the cavity and exciton modes:

$$\delta_{\mathbf{k}} \equiv E_{\gamma}(\mathbf{k}) - E_{\mathrm{ex}}(\mathbf{k}). \tag{10}$$

The *Hopfield coefficients* used to diagonalize this Hamiltonian are most simply expressed as a function of the upper polariton dispersion relation (9):

$$C_{\mathbf{k}} = \frac{\hbar \Omega_{\mathrm{R}}}{\sqrt{(E_{\mathrm{U}}(\mathbf{k}) - E_{\mathrm{ex}}(\mathbf{k}))^2 + \hbar^2 \Omega_{\mathrm{R}}^2}}, \tag{11}$$

$$X_{\mathbf{k}} = \frac{E_{\mathrm{U}}(\mathbf{k}) - E_{\mathrm{ex}}(\mathbf{k})}{\sqrt{(E_{\mathrm{U}}(\mathbf{k}) - E_{\mathrm{ex}}(\mathbf{k}))^2 + \hbar^2 \Omega_{\mathrm{R}}^2}}. \tag{12}$$

Equation (9) is one of the major results of microcavity polariton physics due to the various consequences this relation bears on many key issues that we are going to address in the following sections. It is plotted as solid curves in figure 1, where the dispersion for the exciton, equation (3), and the photon, equation (4), are also plotted in dashed curves. Measurements of angle resolved photoluminescence (PL), e.g. [15], confirm this theoretical description. As the result of the interaction, there is an avoided crossing (anticrossing) of energies. The polariton therefore arises as a coherent mixture of the photon and exciton states. It is the true eigenstate of the system, whereas photon and exciton modes are transient states, exchanging the energy at the Rabi frequency Ω_{R}. The Hopfield coefficients (6) are the fractions of the exciton or photon part of the polariton. In this simple picture the anticrossing appears however weak the interaction is. This can be made more realistic, taking into account the broadening of exciton and photon resonances, by adding to equations (3) and (4) imaginary components $-\mathrm{i}\Gamma_{\mathrm{ex}}$ and $-\mathrm{i}\Gamma_{\gamma}$ respectively. Γ_{ex} is the broadening caused by exciton interactions (inter-particles or with phonons), while Γ_{γ} reflects the finite reflectivity which is inversely proportional to the quality factor of the cavity. At zero detuning equation (9) then becomes

$$E_{\substack{\mathrm{U} \\ \mathrm{L}}}(\mathbf{k}) = \tfrac{1}{2}(E_{\mathrm{ex}}(\mathbf{k}) + E_{\gamma}(\mathbf{k}) - \mathrm{i}\Gamma_{\mathrm{ex}} - \mathrm{i}\Gamma_{\gamma}) \pm \tfrac{1}{2}\sqrt{\hbar^2 \Omega_{\mathrm{R}}^2 - (\Gamma_{\mathrm{ex}} - \Gamma_{\gamma})^2}. \tag{13}$$

The consequences of this equation depend on the sign of the expression under the radical, demonstrating that the physical behaviour of the system depends on the interrelation between

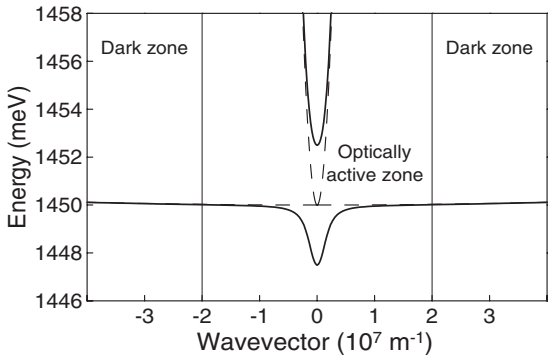

Figure 1. Dispersion relations of a typical GaAs-based cavity. Dashed curves show the bare photon mode dispersion (nearly parabolic because of the confinement) and exciton mode. Solid curves are the new eigenmodes in strong coupling, termed upper-branch and lower-branch polaritons. Polariton laser effect pertains to lower-branch polaritons.

the strength of the exciton–photon coupling and dissipation. If $\hbar\Omega_R > (\Gamma_{ex} - \Gamma_\gamma)$, $E_{\substack{U\\L}}$ exhibits the energy splitting already encountered, which corresponds to the so-called *strong coupling* regime, where the correlations between exciton and photon are important and their interaction cannot be dealt with in a perturbative way. An altogether new behaviour of the system is expected and should be described in terms of polaritons. Weisbuch *et al* [16] were the first to experimentally observe this regime, manifested in the splitting of the reflectivity peaks about the cavity resonant frequency. However, if $\hbar\Omega_R < (\Gamma_{ex} - \Gamma_\gamma)$, the square root becomes imaginary and thus the (real) energy anticrossing disappears and the energy broadening (imaginary part of (13)) becomes split. This is the *weak coupling* regime where the system can be described in terms of weakly interacting photons and excitons. More detailed analysis [13, 14] shows that with the reflectivity going to zero, the broadening of the photon mode diverges (the photon does not remain in the cavity) and the broadening of the exciton mode approaches the bare QW spontaneous emission rate, describing in effect weakly-interacting photons and excitons. In the strong coupling regime, the polaritons (and especially the lower-branch polaritons, which we will exclusively refer to, from now on) have many specific properties endowing the device with revolutionary features. Namely, polaritons are photon-like particles with a very small effective mass which can still scatter efficiently between themselves and with phonons. Their bosonic nature, along with this possibility of efficient relaxation by scattering, allows one to envision interesting dynamics powered by stimulation and therefore leading to the appearance of coherence. It is important to benefit from these specificities to retain the strong coupling regime, which is an experimental difficulty and a technological challenge. Wide bandgap materials such as GaN-based devices, which hold this regime up to room temperature [17], pave the route towards the realization of such devices. Although $\Gamma_{ex} - \Gamma_\gamma$ appears in the criterion of strong coupling from a simplified point of view given by equation (13), experimentally, where this splitting is observed for example by photoluminescence, only the sum of these two broadenings, $\Gamma = \Gamma_{ex} + \Gamma_\gamma$, can be observed. In the special case of equal population of the two branches, the splitting in photoluminescence is derived in [13] as

$$\Omega_{PL} = \sqrt{2\Omega_R\sqrt{\Omega_R^2 + 4\Gamma^2} - \Omega_R^2 - 4\Gamma^2}, \tag{14}$$

so that it is possible that even in the strong coupling regime (where the device would exhibit effects expected from polaritons), the splitting cannot be resolved from PL observations.

Before we address the description of polariton dynamics in the next section, let us briefly discuss the question of interparticle interactions, namely, polariton–polariton interactions. Polaritons can scatter with each other thanks to their underlying excitonic fraction, of the dipole–dipole interaction type, so that they are weakly interacting. The theory previously discussed pertains to the linear—or noninteracting—regime, and is universally accepted as correct. The nonlinear theory however finds conflicting coverage in the literature, as is most clearly illustrated by the work of Combescot *et al* [18] who show that the composite nature of polaritons (or excitons) prevents one from using an effective interacting Hamiltonian of the usual type (for elementary particles) $\frac{1}{2}\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\mathbf{q}} a^{\dagger}_{\mathbf{k}+\mathbf{q}} a^{\dagger}_{\mathbf{k}'-\mathbf{q}} a_{\mathbf{k}} a_{\mathbf{k}'}$, with $V_{\mathbf{q}}$ the Fourier transform of the interaction potential. The authors advocate instead the recourse to a formalism they develop, the *close to boson* formalism. In this paper, we take the view that the predominant issues of a polariton device arise from the interplay of favourable factors (stimulation, small mass, delocalization, density of final states, polariton trap, etc) against competing factors (bottleneck, short lifetime, etc). We believe that the problem is essentially dynamical and regard the finite lifetime as more important than deviations from exact bosonic algebra, or similar issues held in the steady state. We therefore make the approximation that polaritons are true bosons at the densities of interest, and that the specificity of microcavities essentially comes from the dispersion relation, the dimensionality, and the finite and short lifetime. In what follows, based on these approximations, we develop the kinetic equation describing quantities of interest, namely populations and coherence degree, and numerically solve it with parameters modelling realistic cavities. We recommend [18] or [19] for treatments giving full credit to the underlying fermionic structures.

2.2. *Semi-classical Boltzmann description: stimulation and bottleneck*

The simplest, yet to date the most fruitful, formalism to describe the dynamics of polaritons is the semi-classical Boltzmann equation, which reads in momentum space

$$\frac{\mathrm{d}n_{\mathbf{k}}}{\mathrm{d}t} = (1+n_{\mathbf{k}})\sum_{\mathbf{q}\neq\mathbf{k}} W_{\mathbf{q}\to\mathbf{k}} n_{\mathbf{q}} - n_{\mathbf{k}}\sum_{\mathbf{q}\neq\mathbf{k}} W_{\mathbf{k}\to\mathbf{q}}(1+n_{\mathbf{q}}) + P_{\mathbf{k}} - \frac{n_{\mathbf{k}}}{\tau_{\mathbf{k}}}, \tag{15}$$

where $n_{\mathbf{k}}$ is the average number of polaritons with wavevector $\mathbf{k}$, $P_{\mathbf{k}}$ is an external source term (a pump), $1/\tau_{\mathbf{k}}$ is the decay rate (radiative and nonradiative) for a polariton in this state, and $W_{\mathbf{k}\to\mathbf{q}}$ is the rate of transitions from state $\mathbf{k}$ to state $\mathbf{q}$. These transitions can be caused by several mechanisms, but especially the phonon-mediated scattering and the polariton–polariton scattering. Matrix elements for these processes have been calculated by Tassone [20] (but see [18] for the polariton–polariton term). Also one can add, to ease the relaxation, the scattering between polaritons and a gas of cold residual electrons [21]. However, it should be noted that, as far as the Boltzmann equation is concerned, the actual decomposition of these transition rates is relevant only when it comes to simulating a realistic structure, and that otherwise there is no fundamental difference between a relaxation involving a phonon or elastic scattering with another polariton. If an effect can be evidenced with equation (15) it should never be attributed to one specific mechanism; for example, one cannot claim that polariton–polariton interactions are intrinsically required, since identical results can be achieved dispensing from interparticle interactions by merely promoting another relaxation mechanism (e.g., diffusion with free electrons) to enhance the transition rates.

Equation (15) along with the characteristic dispersion relation of polaritons (equation (9) and figure 1) and with suitable expressions for parameters $W_{\mathbf{k}\to\mathbf{k}'}$ and $\tau_{\mathbf{k}}$, has proved successful in providing quantitative agreement with photoluminescence experiments. It already contains in effect most of the key ingredients of polariton physics. There is the stimulation which

is embedded in the $(1 + n_{\mathbf{k}})$ expression, which helps the accumulation of polaritons in the ground state. For all other states, which are degenerate in energy and extend on a so-called elastic circle, the polaritons redistribute themselves very quickly so that the average number of particles per states is reduced considerably, fighting against stimulated effects which are due to high occupancy of excited states. On the other end, the radiative decay rate $1/\tau_{\mathbf{k}}$ increases with $|\mathbf{k}|$ approaching zero because the photon fraction is higher in this case and the probability of radiative escape increases in proportion. This requires efficient relaxation since a polariton does not remain for long in this part of the dispersion where it can experience scattering that will bring it into the ground state. The Hopfield coefficients (equation (6)) make the polariton photon-like at the inflection point of the dispersion relation. Finally, the momentum–energy conservation requirement which must be satisfied for the Boltzmann equation is dictated by the shape of the dispersion. Its highly unusual nature coupled to its two-dimensional character add further complication to the relaxation. The point where two polaritons can scatter through a momentum–energy conserving process with one of them falling in the ground state is referred to as the *magic angle* because of the outstanding effect revealed by pump–probe experiments [4] where this point was excited resonantly. The dramatic enhancement of diffusion when the ground state was initially populated (by a probe) has been the clearest evidence of boson stimulation in microcavities. However, such important experiments fall in the province of coherent control, since the polaritons created at the magic angle are already coherent and the coherence acquired by the ground state is transferred there rather than created. It is an experimental concern when looking for the polariton laser effect (as opposed to a *parametric amplifier* in the case of a pump–probe experiment) that the excitation is either not resonant or is otherwise in remote enough regions of the dispersion so that the coherence is quickly lost through several scattering events. The combination of these phase-space restrictions with lifetime gives rise to the *bottleneck* effect, i.e., a dynamical barrier which blocks (at the inflection point) the polaritons *en-route* towards the ground state. A structure where the detuning is chosen to match the magic angle with the inflection point would benefit from parametric amplification and drastically favour condensation.

Many results have been obtained in the investigation of the interplay of these competing effects by numerically solving the Boltzmann equation. We refer the reader to [20, 22, 23]. However, a shortcoming of this formalism is that it is essentially classical. The stimulation is a mere renormalization of transition rates and most of the important quantities, first of all the coherence, are out of its scope. In the next section, to remedy this limitation while still keeping the full benefit from a successful Boltzmann picture, we extend the formalism to describe the kinetics of the ground state density matrix.

2.3. Ground state density matrix approach

For realistic structures the Boltzmann equation predicts that a regime can be reached where a ground state population builds up in a way very similar to a conventional Bose condensate, for example, of cold atoms [22, 23]. Deng *et al* [10] have studied the second-order coherence of the ground state population with respect to pumping in order to find traces of the Bose condensation of polaritons. It should be emphasized before going deeper into analysis of the coherence that strictly speaking, Bose condensation as a phase transition is impossible in two-dimensional systems. Possible phase transitions like the Kosterlitz–Thouless transition towards superfluidity [24, 25] would indeed be accompanied by a second-order coherence buildup, but conversely such a buildup is not enough by itself to fully evidence a bosonic phase transition. To obtain an experimental signature of the (quasi-)Bose condensation of polaritons, the experimentalists should search for a self-established phase of light emitted by a polariton

laser. If light is emitted by both ± 1 spin-degenerated polariton states, the buildup of a linear polarization in the emission of polariton lasers would be clear evidence of the order parameter appearance in the system. The second-order coherence can be exhibited by a system having no order parameter. Nevertheless, it is an important characteristic of laser light and it merits a careful experimental and theoretical investigation. In this section we present the master equation for the ground state density matrix ρ_0 and its solution, which allow the analysis of the second-order coherence in polariton lasers. Further details can be found in [25–27].

In what follows we keep the basis of plane waves with appropriate boundary conditions and denote the Fock state with $n_{\mathbf{k}}$ polaritons by $|n_{\mathbf{k}}\rangle$, with associated creation/annihilation operators $a^\dagger_{\mathbf{k}}$, $a_{\mathbf{k}}$, dropping the superscript L since we shall consider only the lower branch. In particular, $a^\dagger_0$, a_0 are the boson operators for the ground state. We denote ρ the density matrix for the entire system (comprising polaritons of all momenta but also phonons and hypothetically free electrons), and ρ_0 the ground state reduced density matrix, i.e., $\rho_0 \equiv \mathrm{Tr}(\rho)$ where the trace is taken over all quantum degrees of freedom but for the polariton ground state.

In the Born–Markov approximation one can derive from a microscopic Hamiltonian [25] an equation of motion for ρ_0. The Born part of this approximation grants the density matrix as approximately satisfying for all times the following factorization:

$$\rho \approx \rho_0 \rho_{\mathrm{pol}} \rho_{\mathrm{ph}} \rho_{\mathrm{el}}, \tag{16}$$

where ρ_{pol}, ρ_{ph} and ρ_{el} are the density matrices for the polaritons in excited states, phonons, and electrons (if considered) respectively. Doing so we neglect the correlations between the different quantum states associated with the respective density matrix. In particular, we neglect the correlations between the number of polaritons in the ground state and the number of polaritons of a given excited state. In the same spirit all anomalous correlations, i.e., those which have no interpretation in terms of classical quantities (particle number), are neglected. Also the infinite heat capacity approximation is made for ρ_{ph} and ρ_{el}, i.e., these are made time-independent and diagonal with Bose–Einstein and Fermi–Dirac distributions respectively. As for the Markov approximation, it demands a slow time evolution as compared to the typical collision times. The resulting master equation reads

$$\dot\rho_0 = -\tfrac{1}{2}[R_{\mathrm{out}}(t)(a^\dagger_0 a_0 \rho_0 - 2a_0\rho_0 a^\dagger_0 + \rho_0 a^\dagger_0 a_0) + R_{\mathrm{in}}(t)(a_0 a^\dagger_0 \rho_0 - 2a^\dagger_0 \rho_0 a_0 + \rho_0 a_0 a^\dagger_0)], \tag{17}$$

where R_{out} and R_{in} are time-dependent parameters obtained from the microscopic distributions [25] as $R_{\mathrm{in}} \equiv \sum_{\mathbf{q}\neq 0} W_{\mathbf{q}\to\mathbf{0}} n_{\mathbf{q}}$ and $R_{\mathrm{out}} \equiv \sum_{\mathbf{q}\neq 0} W_{\mathbf{0}\to\mathbf{q}}(1+n_{\mathbf{q}}) - \frac{1}{\tau_{\mathbf{0}}}$. Note that the lifetime is contained in R_{out}. They turn out to be exactly (in the approximations listed above) the rate transitions of the semi-classical Boltzmann equation (15). These rates are plotted in a special case favourable to condensation in figure (2).

In what concerns the average condensate population $n_0 \equiv \mathrm{Tr}(\rho_0 a^\dagger_0 a_0)$ this description provides the same dynamics as the Boltzmann equation (see equation (15)). Namely,

$$\dot n_0 = (1+n_0)\sum_{\mathbf{q}\neq\mathbf{0}} W_{\mathbf{q}\to\mathbf{0}} n_{\mathbf{q}} - n_0 \sum_{\mathbf{q}\neq\mathbf{0}} W_{\mathbf{0}\to\mathbf{q}}(1+n_{\mathbf{q}}) - \frac{n_0}{\tau_{\mathbf{0}}}. \tag{18}$$

However, equation (17) allows access to quantities previously irrelevant to the formalism, like the order parameter $\langle a_0\rangle$ or the coherence degrees of various orders. In particular, the second-order coherence is defined as [10, 28]

$$g^{(2)}(t') \equiv \frac{\langle a^\dagger_0(t) a^\dagger_0(t+t') a_0(t+t') a_0(t)\rangle}{\langle a^\dagger_0(t) a_0(t)\rangle \langle a^\dagger_0(t+t') a_0(t+t')\rangle}. \tag{19}$$

The first-order coherence, $g^{(1)}(t') \equiv \langle a^\dagger_0(t+t') a_0(t)\rangle / \left(\langle a^\dagger_0(t+t') a_0(t+t')\rangle \langle a^\dagger_0(t) a_0(t)\rangle\right)^{1/2}$, is less important for a single-mode where it is largely independent of the structure of ρ_0. In both

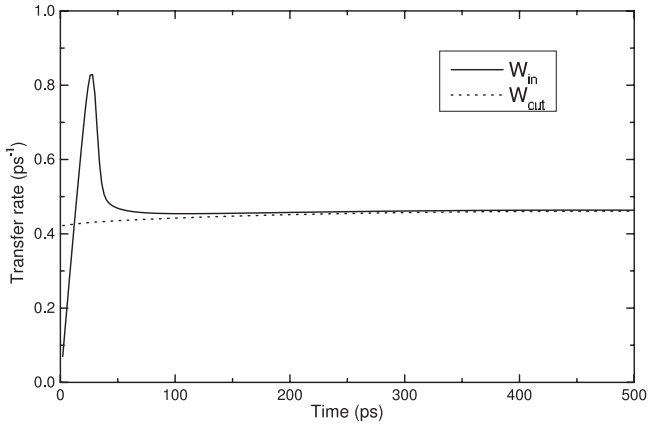

Figure 2. Transition rates $R_{\rm in}$, $R_{\rm out}$ as a function of time, as computed numerically from the semi-classical Boltzmann equation for a typical GaAs-based cavity and for a strong enough pumping, so that $R_{\rm in}$ overcomes significantly $R_{\rm out}$ at an early relaxation stage. In the infinite time limit, $R_{\rm in} < R_{\rm out}$ and all the coherence is lost in the system, but can be retained over macroscopic times. At lower pumping, $R_{\rm in}$ always remains less than $R_{\rm out}$ and the coherence injected into the system is quickly lost.

cases, the t' dependence can be computed from knowledge of the single-time density matrix with the quantum regression theorem, but we shall not use it here, and restrict the consideration to $g^{(2)}(0)$ (see however [23]). This zero-delay, second-order coherence degree is a quantity sufficient to evidence coherence (see, e.g., [29] from the theoretical side and [10] for an experimental application in our context). It is convenient to define a normalized *coherence degree* η by

$$\eta \equiv 2 - g^{(2)}(0), \tag{20}$$

so that $\eta = 0$ for a thermal state, i.e., a state without coherence, and $\eta = 1$ for a coherent state [29]. This parameter can be ill-behaved, as, for example, for the Fock state $|n\rangle$ where $\eta = 1 + 1/n$, but we shall see that for a significant gamut of states it can indeed be interpreted in terms of a ratio of coherence.

With equation (17) one gets the following equation of motion for the 'order parameter':

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle a_0\rangle = \frac{1}{2}(R_{\rm in} - R_{\rm out})\langle a_0\rangle. \tag{21}$$

The equation for $\langle a_0\rangle$ is homogeneous, and, independently of the evolution of n_0, the coherence cannot appear in the system if it is entirely absent at initial time, i.e., if $\langle a_0\rangle = 0$ at $t = 0$. There is amplification thanks to stimulation which builds up a population of higher steady occupancy with higher pumping power, but in all cases the quantum state grows out of the vacuum as a thermal state whose distribution function flattens to accommodate an increasing number of particles. This result is linked to the well-known inability of the Boltzmann equation to initiate the order parameter formation.

While the master equation cannot describe the spontaneous coherence buildup, it is valid to describe the dynamics between the condensate and its 'vapour'. If we simulate the condensate formation with a coherent seed as an initial condition, it is possible to derive the analytical solution for $\rho_0(t)$ (see [26] for details). For the initial condition $\rho_0(0) = |\alpha_0\rangle\langle\alpha_0|$, with $|\alpha_0\rangle$ being a coherent state, the solution is

$$\rho_0(t) = \frac{1}{1+m(t)} D(G(t)\alpha_0) \exp\left\{-\ln\left[\frac{1+m(t)}{m(t)}\right] a_0^\dagger a_0\right\} D(G(t)\alpha_0)^\dagger, \tag{22}$$

where $D(\alpha)$ is the displacement operator defined by $D(\alpha) \equiv \exp\{\alpha a_0^\dagger - \alpha^* a_0\}$ and

$$G(t) \equiv \exp\left[\tfrac{1}{2}\int_0^t (R_{\rm in}(\tau) - R_{\rm out}(\tau))\,{\rm d}\tau\right], \qquad m(t) \equiv G(t)^2 \int_0^t \frac{R_{\rm in}(\tau)}{G(\tau)^2}\,{\rm d}\tau. \tag{23}$$

The significance of all these parameters is more transparent in the Glauber–Sudarshan representation of the density matrix, defined by [29]:

$$\rho_0(t) \equiv \int P(\alpha, \alpha^*, t)\,|\alpha\rangle\,\langle\alpha|\,{\rm d}^2\alpha, \tag{24}$$

with α, α^* granted as independent complex variables. In this phase–amplitude representation, the density matrix is

$$P(\alpha, \alpha^*, t) = \frac{1}{\pi m(t)} \exp\left[-\frac{|\alpha - G(t)\alpha_0|^2}{m(t)}\right]. \tag{25}$$

This is the convolution of a Gaussian of variance $m(t)$ with a delta function centred about $G(t)\alpha_0$. The former, as a P function, describes a thermal state of mean $m(t)$, while the latter describes a coherent state of mean $G(t)\alpha_0$. Convolutions of P functions describe superpositions of fields [29] so that we have now completed the description of the dynamics of a coherent seed (which can be profitably seen as an input to a device) in terms of a coherent amplification of this signal, given by the gain $G(t)$ and an incoherent fraction which comes from spontaneous diffusion of the order parameter. This also permits the interpretation of η as the squared coherent fraction of polaritons, since for such a field, which is a superposition of n coherent polaritons with m thermal (or incoherent) polaritons, η evaluates to

$$\eta = \left(\frac{n}{n+m}\right)^2. \tag{26}$$

Exploitation of this formalism requires numerical simulations (needed to calculate the rates in equation (17)) which are discussed in [26]. There it is found that if a pumping threshold is attained, the signal can retain its shape, especially its phase, over macroscopic time duration. Otherwise it gets quickly damped towards a thermal state again, which is being amplified incoherently.

Note that the specifics of the master equation (17) come from the time dependence of rates $R_{\rm in}$ and $R_{\rm out}$, which are being computed from the Boltzmann equation. The instability associated to the sign of $R_{\rm in} - R_{\rm out}$, patent in equation (21), can also motivate this seed as arising from a fluctuation of a classical nature. The appearance of the seed as a result of such fluctuation needs, however, an additional investigation.

3. The onset of coherence

3.1. The need for nonlinearity

The spontaneous coherence buildup is closely related to the correlations omitted by the disentangling of the density matrix given by equation (16), so that the previous formalism is unable to describe the growth of coherence out of the vacuum. In the literature, equation (17) is known as describing the single-mode linear amplifier (but with time-independent coefficients). This is a system where the noise coming from the spontaneous diffusion of polaritons into the ground state (which randomize the phase while increasing the amplitude) is amplified as well as the signal. A laser, however, adds a resonator to the amplifier, or more specifically, an oscillator, which feeds back into the system part of its output. This nonlinearity results in smothering of the noise instead of amplifying it. Such nonlinearities that have been neglected

should now be taken into account. This will dispense from the arbitrariness of a seed, and reconcile our picture with the common view that the Boltzmann equation breaks down at the onset of coherence though it retains its validity on both sides of this stage. As we could expect, not just any kind of nonlinear term that can be supplemented to the Boltzmann equation will bring the desired effect. For instance adding polariton–polariton interactions in the condensate in (17) is of no use in that regard. It amounts to adding a term proportional to $[a_0^{\dagger 2}a_0^2, \rho]$ which one can readily check is unable to grow coherence.

In [27] all nonlinear terms which can be added to (17) to allow for coherence buildup without more involved extensions of the formalism have been systematically tracked down. None of them appears as of special relevance in the case of a polariton laser, but instead they arise in conventional laser theories where they come from the resonator. In our case there is the Fabry–Perot cavity, but it has vanished in the strong coupling regime to allow the formation of the new eigenstates which are polaritons. In the weak coupling regime it is however possible to exhibit an exciton laser of a conventional type with the cavity being the resonator.

In [30] the formalism of the previous section has be reformulated to describe the polariton order parameter evolution, and a general type of nonlinearity has been added to study the coherence buildup without the need of a seed. This description predicts the growing of a randomly phased coherent state of condensed polaritons. The microscopic origin of nonlinearity lies in the correlations between the polaritons in the ground and excited states, taking place due to polariton–polariton repulsion and omitted in the derivation of equation (17).

We expect that two types of nonlinearities may lead to the spontaneous buildup of the coherence in polariton lasers. First, at the high density limit, repulsing interactions between polaritons in the condensate may lead to its depletion manifesting itself through the increase of R_{out}, as discussed in [30]. Second, at lower concentrations of polaritons, the nonlinearity may arise due to correlations between momentary occupation numbers of the ground and excited states that arise if the total number of polaritons in the system is fixed by the excitation conditions. This second mechanism influences both R_{in} and R_{out}. We will not describe the evolution of the whole density matrix of the polariton subsystem, but study the dynamics of its diagonal elements only, in the known framework of the so-called quantum Boltzmann master equation.

3.2. Quantum Boltzmann master equation: the two-oscillators model

Gardiner *et al* [31] derived a quantum kinetic equation successfully describing the growth of a BEC for conserved bosons, at equilibrium. By 'conserved' it is meant that, unlike gauge bosons such as photons or phonons, the particles cannot be simply created or annihilated by the system to adjust to external parameters (typically the temperature). This property is at the heart of BEC since consequently bosons must diffuse to lower energy states until the phase space cannot accommodate all of them, effectively gathering the excess fraction into the ground state. Various approximations conducted on that general kinetic equation yield back famous transport equations, among them the semi-classical Boltzmann equation as the one with the most approximations, of the kind we already discussed and used in [25]. Also here this equation is shown to be invalid for describing the dynamics when the condensate is forming. The fewest approximations that one can relax to obtain a kinetic equation able to grow coherence spontaneously is, in Gardiner's terminology, a *Quantum Boltzmann Master Equation* (QBME). It is an equation with structure that closely follows that of the Boltzmann equation itself—in particular it still depends crucially on the same Boltzmann rates R_{in}, R_{out}—but now instead of being an equation for the distribution function $n_{\mathbf{k}}$, it is an equation for the probability to have configurations with these populations (whence the name 'master equation').

Stated another way, the populations in the semi-classical Boltzmann equation are averages, which we shall thus now write $\langle n_k \rangle$, whereas in the QBME, n_k is a random variable with fluctuations and correlations to other likewise random variables which are not neglected (that is we do not allow such equalities as $\langle n_k n_q \rangle = \langle n_k \rangle \langle n_q \rangle$).

Gardiner *et al* have simulated the exact QBME for cold atoms [32], with impressive agreement with experiment and thermodynamic theories (there obtained from the infinite time limit of kinetic equations). This however calls for heavy numerical Monte Carlo simulations. We shall investigate this equation in the case of a toy model of two oscillators, in which case the QBME is an equation for $p(n, m)$: the probability to have n polaritons in the ground state and m polaritons in the other (excited) state. For simplicity we first ignore pumping and consider infinite lifetime. For relaxations through coupling to a thermal bath which temperature defines transition rates $w_{1\to 2}$ from state 1 to 2 and $w_{2\to 1}$ from 2 to 1 this equation reads

$$\begin{aligned}\dot{p}(n, m) = {} & (n+1)m[w_{1\to 2}p(n+1, m-1) - w_{2\to 1}p(n, m)] \\ & + n(m+1)[w_{2\to 1}p(n-1, m+1) - w_{1\to 2}p(n, m)].\end{aligned} \tag{27}$$

It can be solved exactly in the steady state:

$$p(n, m) = \frac{\xi - 1}{\xi^{n+m+1} - 1}\xi^n P(n+m), \tag{28}$$

with $\xi \equiv w_{2\to 1}/w_{1\to 2}$ (note that by definition of the ground state, $\xi > 1$) and $P(N) \equiv \sum_{n+m=N} p(n, m)$ is the distribution of *total* particle number, i.e., the probability to have N particles in the *entire* system. $P(N)$ is time independent because the relaxation mechanism conserves the particle number. Its definition therefore provides initial conditions. This conservation implies correlations of particle numbers of both states, with full correlations if the number of particles in the whole system is known precisely (in which case $P(N) = \delta_{N,N_0}$). We identified such correlations as responsible for coherence buildup. Indeed we proved that neglecting correlations always yields two thermal states, whereas taking them into account allows, by lowering the temperature (increasing ξ), an initial configuration of two thermal states to reach a configuration where the ground state has acquired some coherence [28]. The criterion of the buildup of coherence in the ground state is that the statistics of the ground state $p_1(n) \equiv \sum_m p(n, m)$ behaves as $P(n)$. This is indeed the case for (28) in the limit of $\xi \to \infty$ (vanishing temperatures). Physically this implies fast relaxation of polaritons into the ground state, when the distribution function reproduces the distribution function of the total number of polaritons in the system. Therefore, if the fluctuations of the total number of polaritons are weak, the fluctuations of the number of polaritons in the ground state are weak as well, and they are second-order coherent.

We have also developed a dynamical description of a simple polariton laser by considering the coherence formation for finite lifetime τ in the presence of pumping (to maintain nonzero steady state population), still in the two-oscillators model. In this case we study p_1, the statistics of the ground state alone, and $\langle m \rangle_n$, the mean number of particles in excited states given there are n in the ground state. Those quantities are linked in the defining formula $\langle m \rangle_n p_1(n) = \sum_m m p(n, m)$. The Boltzmann Master Equation thus reads [28]

$$\begin{aligned}\dot{p}_1(n) = {} & (n+1)\big(w_{1\to 2}(\langle m \rangle_{n+1} + 1) + 1/\tau\big)p_1(n+1) \\ & - \{n(w_{1\to 2}(\langle m \rangle_n + 1) + 1/\tau) + (n+1)w_{2\to 1}\langle m \rangle_n\}\, p_1(n) \\ & + n w_{2\to 1}\langle m \rangle_{n-1} p_1(n-1),\end{aligned} \tag{29}$$

with the steady state solution obtained by detailed balance:

$$p_1(n+1) = \frac{w_{2\to 1}\langle m \rangle_n}{w_{1\to 2}(\langle m \rangle_{n+1} + 1) + 1/\tau} p_1(n). \tag{30}$$

With finite lifetime and pumping, correlations due to the conservation of particle number can only be ascertained in the mean:

$$\langle m \rangle_n = N - n, \tag{31}$$

with N the average total number of particles. Equations (30), (31) and $p_1(1) > p_1(0)$ as the criterion for coherence yield the critical relationship between density N, lifetime τ and temperature $T \propto 1/\ln(w_{1\to 2}/w_{2\to 1})$, for coherence in the steady state:

$$N > \frac{1}{\tau(w_{2\to 1} - w_{1\to 2})}. \tag{32}$$

Equation (29) has been solved numerically for two sets of parameters discriminating the criterion (32), the results of which are plotted in figure 3.

The previous toy model has the merit to delineate the mechanism of coherence buildup and provide some analytical insights. In the next section we extend this procedure to an infinite set of oscillators, modelling the different polariton states.

3.3. Polariton laser case, expressions for the correlations

Although the correlations which will invalidate the Boltzmann equation in this case—and require the use of its master equation instead—might have many conceivable origins, among which interparticle interactions undoubtedly rank as a leading factor, we find that correlations caused by particle number conservation are enough to describe coherence buildup of the noninteracting gas in contact with a thermal bath. Therefore we choose to model the polariton laser effect for a noninteracting polariton gas in the canonical ensemble.

Since we remain interested in the statistics of the ground state only, the kinetic equation is for $p(n_0)$, the probability to have n_0 polaritons in the ground state, and is of a simple rate equation character:

$$\begin{aligned}\dot{p}(n_0) = {} & W_{\text{in}}^{n_0} n_0 p(n_0 - 1) + (W_{\text{out}}^{n_0+1} + 1/\tau_0)(n_0 + 1) p(n_0 + 1) \\ & - W_{\text{in}}^{n_0}(n_0 + 1) p(n_0) - (W_{\text{out}}^{n_0+1} + 1/\tau_0) n_0 p(n_0),\end{aligned} \tag{33}$$

with τ_0 the lifetime of polaritons in the ground state and $W_{\text{in}}^{n_0}$, $W_{\text{out}}^{n_0}$ the incoming and outgoing scattering rates for the ground state *when* it contains n_0 particles. Note the dependence on the ground state occupancy, which appears in the usual rate transitions as a dependency on the number of particles in excited states:

$$W_{\text{in}}^{n_0}(t) = \sum_{\mathbf{q}} W_{\mathbf{q}\to\mathbf{0}}(t) n_{\mathbf{q}}^{n_0}(t), \tag{34}$$

$$W_{\text{out}}^{n_0}(t) = \sum_{\mathbf{q}} W_{\mathbf{0}\to\mathbf{q}}(t)(1 + n_{\mathbf{q}}^{n_0}(t)). \tag{35}$$

The Boltzmann equation deals with mean values, whilst the master equations deal with random variables. We do not want to solve a complete set of master equations for all quantum states in the system as it is extremely heavy from the point of view of numerical calculations and does not allow one to obtain an analytical result. Our method will consist of solving the master equation for the ground state only, taking into account the correlations between n_0 and average occupation numbers of excited states, $n_{\mathbf{q}}$, imposed by the conservation of the number of particles in the system. This approach is valid if the timescale of fluctuations of n_0 is much slower than that of the transitions between the excited states. In this case, one can do the time averaging of fluctuating populations on the scale of fluctuations of n_0. This assumption seems to be quite reasonable bearing in mind that, for the most part, excited states are situated at the exciton part of the lower polariton branch, and transitions between them are governed by fast relaxation processes involving acoustic phonons. On the other hand, the exchange with

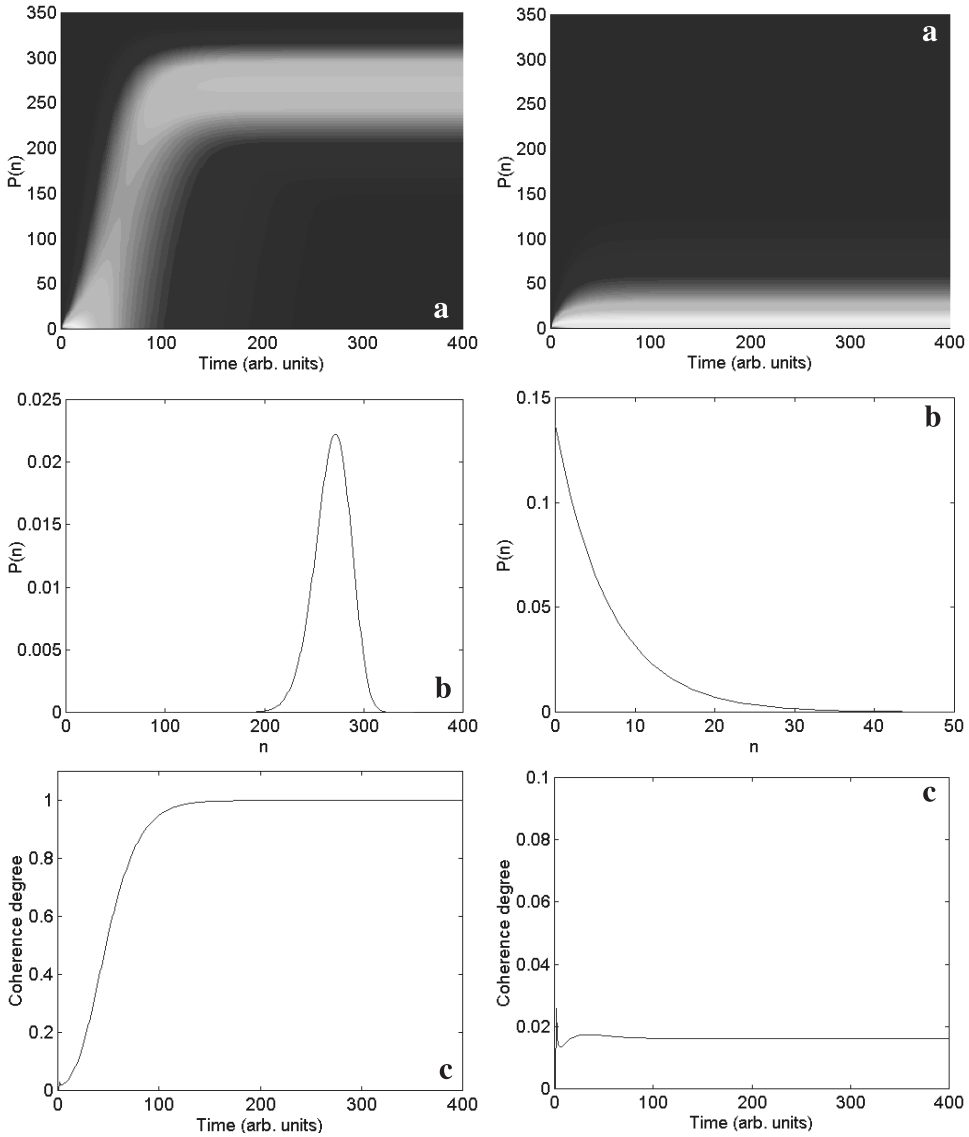

Figure 3. The initial condition is vacuum and parameters are $N = 350$, $w_{1\to 2} = 0.75 \times 10^{-5}$ (arb. units) and $1/\tau = 20 \times 10^{-5}$ (arb. units) for both columns, but the rhs column is for $w_{2\to 1} = 0.95 \times 10^{-5}$ (arb. units) whereas the lhs is for $w_{2\to 1} = 10^{-5}$ (arb. units). All arbitrary units have the same dimension of an inverse time. (a) is the density plot of $p_1(n)$ as a function of time (lighter colours for higher values with black for zero), (b) is $p_1(n)$—a projection of (a)—in the steady state region and (c) is the coherence degree $\eta = 2 - g^{(2)}(0)$. On the lhs the threshold is attained and coherence appears in the system. On the rhs only a thermal state is grown; the coherence remains low.

the ground state is slower because of the phonon bottleneck. The average populations $n_{\mathbf{q}}^{n_0}$ can be obtained from the Boltzmann equation with an additional constraint imposed by the total number of polaritons at the excited states, N_{ex}:

$$N_{\mathrm{ex}} = N - n_0, \tag{36}$$

where N is the total number of polaritons in the system.

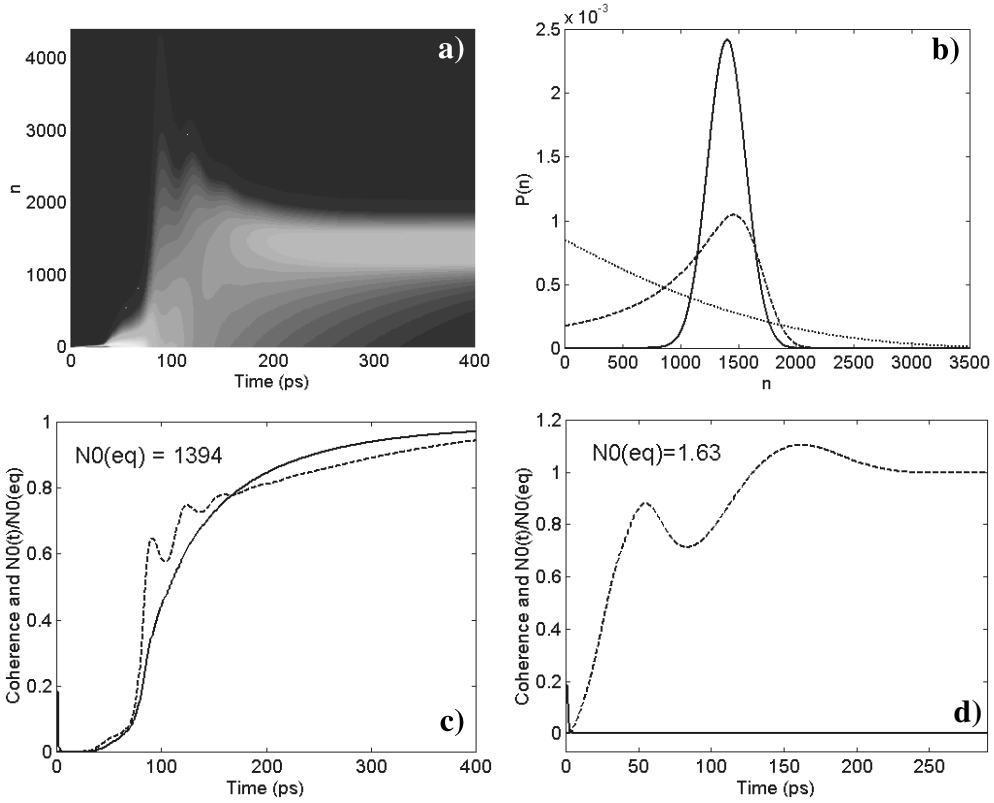

Figure 4. (a) Density plot of the ground state statistic $p(n)$ as a function of time for a pumping power of 500 W cm^{-2} starting from vacuum (cf figure 3) and (b) three projections for various times: the dotted curve at 90 ps has the statistics of a thermal state while the solid curve at 600 ps has the statistics of a coherent state. The dashed curve's statistics at intermediary time of 200 ps sits in between. (c) Coherence degree $\eta = 2 - g^{(2)}(0)$ of the polariton ground state versus time (solid) and ground state population normalized by the value achieved in the steady state (dotted). (d) The same as (c), but for a pumping power four times smaller. In the latter case the ground state population remains small and the coherence degree is zero.

Let us note at this point that the method we used to take into account the correlations between the ground state population and occupation numbers of the excited states, though oversimplified, is qualitatively correct as it reflects the general tendency: the more polaritons are in the ground state, the less of them remain in excited states, if N is conserved. Clearly, in the case of a two-level system this approach coincides with the exact solution described in the previous section.

Equation (33) can now be solved numerically and its second-order coherence degree computed. Results of simulations are plotted in figure 4. The polariton distribution function versus time and scattering rates (34) are obtained by solving the semi-classical Boltzmann equation, as described in [21]. The ground state statistics versus time is calculated simultaneously and self-consistently using the QBME equation (33). Parameters used were for a CdTe microcavity with eight QWs, a Rabi splitting of 15 meV and at zero detuning. Diffusion was mediated by a bath of phonons at a temperature of 6 K and with a residual gas of electrons of density 10^{11} cm^{-2}. Compared with the similar results obtained in the simple two-oscillators picture, cf figure 3, the case with many oscillators and with rate transitions

computed after a realistic model of a microcavity displays intricate dynamics with complex evolution from thermal to coherent statistics. However, the main features are shared by both systems, validating the two-oscillators system as a good aid in understanding how coherence appears without prior existence in an actual device. The oscillations of N_0 observed at initial times (about 100 ps) are caused by a competing population of ground state and bottleneck in a cavity. With increasing population, the bottleneck is shifted towards lower momenta, until it overlaps the magic angle which results in a collapse of an important fraction of polaritons into the ground state by effective scattering. The bottleneck is therefore strongly and suddenly depleted, emptying excited states that were feeding the ground state. The latter, not being sufficiently provided for anymore, decays in amplitude until the pump replenishes excited states which can stock up in the bottleneck again and start anew populating the ground state. This process repeats until a population large enough to sustain continuously stimulated emission from excited states is reached, so that they are no longer abruptly emptied by collapse into the ground state. Interestingly, although the population is oscillating back and forth at this stage, the coherence steadily increases. In this regime where coherence can build up, starting from vacuum, the state first turns into a thermal state (cf figure 4(b)) with exponentially decreasing statistics, then it starts to develop a nonzero maximum while still being strongly asymmetric, until it eventually displays the statistics of a coherent state. However, in a regime where coherence is unable to build up, as is the case of figure 4(d) where the pumping power is four times smaller than for the previous case, the statistics remains those of a thermal state at all times.

4. Conclusions

We have presented an overview of the physics of microcavity polaritons, introducing key notions and putting emphasis on the existence of a strong coupling regime where new eigenstates of the microcavity (polaritons) are bosons at sufficiently low densities, with appealing characteristics to exhibit coherent phenomena. We studied theoretically the second-order coherence of a condensate of polaritons formed in this strong coupling regime. We showed that the coherence, once introduced in the cavity by a 'seed' population of coherent polaritons, can survive over a macroscopically long time. We have also investigated the possibility of coherence buildup for the polaritons in the ground state without a seed. This effect can be described by introducing the dependence of polariton relaxation rates W_{in} and W_{out} on the population of the ground state (rather than on its average value). This dependence arises from either conservation of the total number of particles in the system that imposes correlations between momentary populations of ground and excited states or from polariton–polariton interaction. Both effects are essentially classical. Spontaneous coherence buildup can indeed be observed, while it is not yet an evidence for polariton Bose condensation or superfluid phase transition.

References

[1] Le Si Dang D, Heger D, André R, Boeuf F and Romestain R 1998 *Phys. Rev. Lett.* **81** 3920
[2] Senellart P and Bloch J 1999 *Phys. Rev. Lett.* **82** 1233
[3] Stevenson R M, Astratov V N, Skolnick M S, Whittaker D M, Emam-Ismail M, Tartakovskii A I, Savvidis P G, Baumberg J J and Roberts J S 2000 *Phys. Rev. Lett.* **85** 3680
[4] Savvidis P G, Baumberg J J, Stevenson R M, Skolnick M S, Whittaker D M and Roberts J S 2000 *Phys. Rev. Lett.* **84** 1547
[5] Boeuf F, André R, Romestain R and Le Si Dang D 2000 *Phys. Rev.* B **62** R2279
[6] Tartakovskii A I, Emam-Ismail M, Stevenson R M, Skolnick M S, Astratov V N, Whittaker D M, Baumberg J J and Roberts J S 2000 *Phys. Rev.* B **62** R2283

[7] Alexandrou A, Bianchi G, Péronned E, Hallé B, Boeuf F, André R, Romestain R and Le Si Dang D 2001 *Phys. Rev.* B **64** 233318
[8] Saba M, Ciuti C, Bloch J, Thierry-Mieg V, André R, Le Si Dang D, Kundermann S, Mura A, Bongiovanni G, Staehli J L and Deveaud B 2001 *Nature* **414** 731
[9] Huang R, Yamamoto Y, André R, Bleuse J, Muller M and Ulmer-Tuffigo H 2002 *Phys. Rev.* B **65** 165314
[10] Deng H, Weihs G, Santori C, Bloch J and Yamamoto Y 2002 *Science* **298** 199
[11] Qarry A, Ramon G, Rapaport R, Cohen E, Ron A, Mann A, Linder E and Pfeiffer L N 2003 *Phys. Rev.* B **67** 115320
[12] Haug H and Kosh S W 1990 *Quantum Theory of the Optical and Electronic Properties of Semiconductors* (Singapore: World Scientific)
[13] Savona V and Tassone F 1995 *Solid State Commun.* **95** 673
[14] Kavokin V and Malpuech G 2003 *Cavity Polaritons* (Amsterdam: Elsevier)
[15] Houdré R, Weisbuch C, Stanley R P, Oesterle U, Pellandin P and Ilegems M 1994 *Phys. Rev. Lett.* **73** 15
[16] Weisbuch C, Nishioka M, Ishikawa A and Arakawa Y 1992 *Phys. Rev. Lett.* **69** 3314
[17] Antoine-Vincent N, Natali F, Byrne D, Vasson A, Disseix P, Leymarie J, Leroux M, Semond F and Massies J 2003 *Phys. Rev.* B **68** 153313
[18] Combescot M and Betbeder-Matibet O 2002 *Europhys. Lett.* **59** 579
[19] Szymanska M H, Littlewood P B and Simons B D 2003 *Phys. Rev.* A **68** 013818
[20] Tassone F and Yamamoto Y 1999 *Phys. Rev.* B **59** 10830
[21] Malpuech G, Kavokin A, Di Carlo A and Baumberg J J 2002 *Phys. Rev.* B **65** 153310
[22] Malpuech G, Di Carlo A, Kavokin A, Baumberg J J, Zamfirescu M and Lugli P 2002 *Appl. Phys. Lett.* **81** 412
[23] Porras D and Tejedor C 2003 *Phys. Rev.* B **67** 161310
[24] Kavokin A, Malpuech G and Laussy F P 2003 *Phys. Lett.* A **306** 187
[25] Malpuech G, Rubo Y G, Laussy F P, Bigenwald P and Kavokin A V 2003 *Semicond. Sci. Technol.* **18** S395
[26] Rubo Y G, Laussy F P, Malpuech G, Kavokin A and Bigenwald P 2003 *Phys. Rev. Lett.* **91** 156403
[27] Laussy F P, Rubo Y G, Malpuech G, Kavokin A and Bigenwald P 2003 *Phys. Status Solidi* c **0** 1476
[28] Laussy F P, Malpuech G and Kavokin A 2004 *Phys. Status Solidi* c **1** 1339
[29] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
[30] Rubo Y G 2004 *Phys. Status Solidi* c **201** 641
[31] Gardiner C W and Zoller P 1997 *Phys. Rev.* A **55** 2902
[32] Jaksch D, Gardiner C W and Zoller P 1997 *Phys. Rev.* A **56** 575